The Relative Performance Puzzle

Ernst Maug[1]
Duke University
Fuqua School of Business
Durham, NC 27708
phone: (919) 660-7753, fax: (919) 684-2818
email: maug@mail.duke.edu
http://www.duke.edu/~maug

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Abstract

The accepted theoretical models of executive compensation contracts all seem to imply that optimal remuneration packages should contain a relative performance element. The puzzle is that the empirical literature has found remarkably little relative performance evaluation. This paper aims at resolving this puzzle by introducing the notion that the manager can trade on assets other than her own company's stock. Then the manager's portfolio strategy always adjusts for the risks of her compensation contract and she replaces the firm's benchmark with a "home-made" benchmark. She chooses exactly the weights and the composition of the benchmark that would otherwise be chosen in an optimal contract. In many cases this is possible without short selling any assets. To the extent that performance benchmarks are correlated with traded assets they are redundant for the optimal contract. Accounting benchmarks are exempt from this verdict since they may help to insure the manager against risks that are not related to traded assets. This may help to understand the presence of relative performance elements in annual bonus plans.

JEL Classification:

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1 Introduction

The empirical literature on executive compensation has found little if any evidence for the use of relative performance compensation, where compensation is assessed as a function of some performance measure relative to a benchmark.[1] This seems to be in stark contradiction to the notion that optimal compensation contracts should make use of all available information. Also, Murphy (1999) documents that even the few companies that make use of relative performance evaluation use it in their annual bonus plans, which are most likely based on accounting profits rather than stock market returns.[2] This paper develops one approach to address this puzzle.

The argument is based on the insight that managers' exposure to risk does not only derive from their compensation contract, but also from all the other assets they hold in their portfolio.[3] The paper shows that if managers can adjust their portfolio holdings optimally, then there is typically no or only a small role for relative performance benchmarks. The reason is that managers evaluate compensation contracts only with respect to the

[1] Jensen and Murphy's (1990b) classic study finds that relative performance evaluation plays only a minor role in executive compensation contracts. These findings are confirmed by other studies by Lambert and Larcker (1987), Garen (1994) and Aggarwal and Samwick (1996), (1999). Some support was found by Antle and Smith (1986) and by Gibbons and Murphy (1990). Contradicting results were found by Barro and Barro (1990). Janakiraman, Lambert and Larcker (1992) find that compensation increases with the industry's stock performance and decreases with the industry's accounting performance, thus providing inconclusive evidence.

[2] Murphy (1998) also notes that most plans use relative performance measures differently from the way prescribed by the model. They use industry rankings and peer comparisons rather than an average of the respective peer group.

[3] A similar observation was made by Lambert, Larcker and Verrecchia (1991).

incremental risk they incur. Their optimal portfolio strategy always reduces exposure to those assets that are highly correlated with the stock of their own company, and increases holdings in any assets positively correlated with the benchmark. Hence, the incremental risk they incur is always only the firm-specific risk, and relative performance evaluation can improve on this only to the extent that the benchmark used in the contract is uncorrelated with traded assets and correlated with the specific-risk of the firm. In most cases, the optimal portfolio adjustment by managers will not include short selling of any assets.

The intuition is simple and best explained by an example. Consider the case of a manager whose current wealth is $1,000,000. Of this $500,000 is invested in an index fund and the remaining $500,000 is invested in cash. Assume the index fund is perfectly correlated with the market portfolio. Now suppose this manager is hired into a company and accepts a contract that forces her to invest $200,000 in restricted stock of the company she manages. Then she will optimally rebalance her portfolio and reduce her holdings in the index fund to $300,000. Therefore, the additional risk she incurs through her new employment is only the specific risk of this company, not the company's total risk. For every additional dollar of restricted stock she needs to hold as part of her contract she reduces her holdings in the index fund by exactly one dollar. When she considers the conditions of the contract she is offered she will take the adjustment to her portfolio into account, and evaluate the contract on the basis of the incremental risk involved in accepting it, not the total risk.

Now, suppose she is offered a relative performance contract, where the performance of her company is benchmarked against the stock market index. This is equivalent to offering her a contract without relative performance features, plus a short position in the stock market portfolio. Then the manager's optimal portfolio allocation will change by neutralizing the relative performance feature with an offsetting long position in the index fund. Hence, the incremental risk she incurs as part of her employment is completely independent of the relative performance elements of her contract.

The argument needs to be modified if the benchmark is not a traded asset like the stock market index, but an accounting benchmark, for example the average earnings of companies in the same industry, or average returns on the book value of assets or the book value of equity. Then the manager can adjust her portfolio only to the extent that the benchmark is correlated with any of the assets she can trade on. Therefore, accounting benchmarks improve the compensation contract to the extent that they are not correlated with any traded assets, and to the extent that they help to reduce the manager's exposure to risks that are not correlated with those of traded assets other than the firm's own stock.

The next section 2 discusses the literature. Section 3 discusses the base model for this analysis and reiterates the argument for relative performance evaluation in an environment where the manager cannot trade. Then section 4 introduces the possibility that the manager can trade on the market index, and section 5 shows how the argument changes if the manager is evaluated relative to an accounting benchmark. Section 6 generalizes the argument by allowing the manager to trade on the components of the market index individually rather than on the index as a whole. Section 7 concludes. Technical derivations and proofs are deferred to the appendix.

2 Discussion of the Literature

Several papers have already analyzed the implications of relative performance evaluation for investment behavior. Scharfstein and Stein (1990) rationalize the "herding" behavior induced by relative performance evaluation, where managers prefer an inferior action because it is taken by the majority of agents they are compared with. In a model with career concerns Zwiebel (1995) shows that mediocre managers may prefer the inferior (conservative) action, whereas good and bad managers prefer superior actions. These papers analyze the implications of relative performance evaluation outside of an optimal contracting model since the relative performance element is the implication of career concerns.[4] Similarly, Aggarwal and Samwick (1996) show that relative performance evaluation may have implications for managers' competitive behavior.

Admati and Pfleiderer (1997) show a related result in the context of delegated portfolio management. However, in their model the principal is risk-averse, and the result concerns mainly the implications of performance benchmarking for optimal risk sharing. Generally, contracting problems in delegated portfolio management are somewhat different from contracting problems in the firm, and only the latter are the focus of this paper.

Another literature has previously looked at the impact of managers trading of their own stock. Neus (1989) and Laux (1990), (1998) derive the trading decisions of an owner-manager who can trade in the shares of the stock of her own company, and show to what extent the standard principal-agent

[4] See also Huddart (1999) for a model of delegated portfolio management with reputation effects. There is a large literature on models with career concerns that is not discussed here.

contract changes dependent on the “undoing” of the contract the agent can undertake in the capital market. Laux also argues that the agent could replicate insurance by the firm with self-insurance in the capital market, but does not show to what extent the optimal trading strategy of the agent coincides with the optimal contract chosen by the principal. However, this is critical in view of Campbell and Kracaw’s (1987) result that the insurance chosen by the manager may be different than that preferred by shareholders as part of the optimal contract. Garvey (1997) finds that incentive contracts dominate capital structure as a bonding device absent “undoing” of these contracts by the manager. This ranking may reverse if the manager can neutralize the impact of the contract through trading. The analysis of these papers complements ours, since the paper here assumes that the manager can never trade in her own stock. Campbell and Kracaw (1985), (1987) analyze models of executive compensation where the manager-agents can also insure herself in the capital market. However, these papers do not analyze the implications for relative performance evaluation. In Campbell’s and Kracaw’s approach many of the considerations here could not be analyzed since the insurable risks are uncorrelated across firms.

3 The Rationale behind Relative Performance Compensation

In this section I rehearse the standard argument that an optimal principal-agent contract between the shareholders of a firm and management should have a relative performance feature.[5] We study the contracting problem in

[5] The standard results on the informativeness principle and the principal-agent contract as discussed here go back to Holmström’s (1979), (1982) seminal papers. The special case

a firm where current shareholders purchase assets with a value A to realize a single end of period cash flow C. The cash flow depends on the effort of the manager e, a random component ε and a fixed exogenous constant c_0 as follows:

$$C = e + (c_0 + \varepsilon) A \tag{1}$$

where ε is distributed normal with zero mean and variance σ^2. The manager receives income Y at the end of the period. The manager's income is determined by the stock price P and by comparison with some benchmark. For concreteness we specify that the benchmark is the return on the market portfolio r_M. In subsequent sections we shall generalize this setup to include accounting benchmarks. The stock price at the end of the period reflects the cash flow net of the manager's income:

$$Y = \hat{\alpha}_0 + \hat{\alpha}_P P - \hat{\alpha}_B r_M \tag{2}$$

$$P = C - Y \tag{3}$$

Note that the return r_M can be interpreted as a payoff to a one-dollar investment in the market index, so the contract specifies that the agent holds a position of $-\hat{\alpha}_B$ dollars in the index. The firm has one share of stock outstanding, so $\hat{\alpha}_P$ measures the number of shares held by the manager. In this section we assume that the stock market index has a random return r_M with an expected value of zero and variance σ_M^2. This effectively assumes that the market risk premium and the risk free rate are both zero. For simplicity we assume that the company under consideration is not part of the market index. These assumptions are inconsequential for the argument presented here.

analyzed here was developed by Holmström and Milgrom (1987).

The cash flow of the firm is imperfectly correlated with the market return. We model this by decomposing the random component of the cash flow, ε, into a component reflecting market risk and a firm-specific component that is uncorrelated with the market return:

$$\begin{aligned} \varepsilon &= \varepsilon_F + r_M \quad \varepsilon_F \sim N\left(0, \sigma_F^2\right) \\ \sigma^2 &= \sigma_F^2 + \sigma_M^2 \end{aligned} \tag{4}$$

It is important to note that the agent has a limited action space in this model since she can only choose her level of effort, and effort affects only the mean of the distribution and not the variance or covariance with the market. If she could also choose the scale of operations A, then she would also influence the riskiness of the operations.[6] Substituting for C from (1) and using $\alpha_P = \frac{\hat{\alpha}_P}{1+\hat{\alpha}_P}$, $\alpha_B = \frac{\hat{\alpha}_B}{1+\hat{\alpha}_P}$ and $\alpha_0 = \frac{\hat{\alpha}_0}{1+\hat{\alpha}_P}$ we can write:

$$Y = \alpha_0 + \alpha_P C - \alpha_B r_M \tag{5}$$

$$P = \frac{C - \hat{\alpha}_0 + \hat{\alpha}_B r_M}{1 + \hat{\alpha}_P} = (1 - \alpha_P) C - \alpha_0 + \alpha_B r_M \tag{6}$$

In this model the manager's wealth W is given only by her income, so $Y = W$. Effort is costly to the manager, and the costs of effort are represented as a deduction of income (or wealth) by a standard quadratic function $\frac{k}{2}e^2$. Then the manager's preferences can be represented by a negative exponential utility function $U(W, e) = -e^{-\rho\left(W - \frac{k}{2}e^2\right)}$. If wealth is normally distributed we can use standard distribution theory to represent the man-

[6] For an analysis where the agent can also adjust investment and influence the riskiness of the firm's operations see Marcus (1982) and Neus (1996).

ager's preferences by the certainty equivalent of her utility as:[7]

$$\begin{aligned} CE(W,e) &= E(W) - \frac{k}{2}e^2 - \frac{\rho}{2}Var(W) \\ &= \alpha_0 + \alpha_P(c_0A + e) - \frac{k}{2}e^2 - \frac{\rho}{2}Var(W) \qquad (7) \\ Var(W) &= \alpha_P^2 A^2 \sigma^2 - 2\alpha_P A \alpha_B \sigma_M^2 + \alpha_B^2 \sigma_M^2 \end{aligned}$$

This gives the manager's first order condition for her choice of the level of effort as $e = \alpha_P/k$. Then standard analysis gives the optimal contract as:[8]

$$\alpha_P^R = \frac{1}{1 + \rho k A^2 \sigma_F^2} \qquad (8)$$

$$\alpha_B^R = \alpha_P A \qquad (9)$$

where the superscript R denotes the optimal contract with relative performance evaluation.[9] Denote by $V = E(P)$ the net value of the firm to shareholders. Then substituting the optimal contract ((8), (9)) back into the shareholder value equation (see equation (35) in the appendix) gives the optimized value for V as:

$$V^R = c_0A + \frac{\alpha_P^R}{2k} \qquad (10)$$

The contract with relative performance evaluation can be easilycontrasted to the standard contract obtained if there is no relative performance element

[7] The certainty equivalent CE is related to the expected utility $EU(W)$ of the manager as $CE(W) = -\log(-EU(W))/\rho$.

[8] A detailed analysis can be found in the appendix. The main steps are to determine that the participation constraint is always binding, so $CE(W,e) = 0$, and that the principal (outside shareholders) maximize the expectation of the firm's net cash flow after deducting the manager's wage.

[9] Our result is similar to Aggarwal and Samwick (1999) (see their section IV) since the stock-beta of our firm is always equal to one and we would need to include the scaling factor A in the definition of ε.

and $\alpha_B = 0$. Then we have:

$$\alpha_P^{NR} = \frac{1}{1 + \rho k A^2 \sigma^2} \tag{11}$$

and the same expression for (10) obtains by simply exchanging α_P^R with α_P^{NR}. Since $\alpha_P^R > \alpha_P^{NR}$, we have conclusively proven the standard result:

Proposition 1 ***(Informativeness principle)*** *An optimal compensation contract always includes a relative performance element. A relative performance contract* $\left(\alpha_0^R,\ \alpha_P^R, \alpha_B^R\right)$ *leads to strictly higher shareholder value* V *and a higher level of effort by the manager than the best attainable outcome without relative performance evaluation* $\left(\alpha_0^{NR},\ \alpha_P^{NR}, 0\right)$.

This result is a special case of Holmström's (1979) informativeness principle which can be rephrased loosely as saying that the optimal contract solves a statistical inference problem. The manager's compensation increases whenever the statistics (performance measures) observed by the principal make it more likely that the manager has provided more effort, and all statistics that contribute additional information to this effect are employed in the optimal contract. Here we see that $C - Ar_M$ is a better statistic for this purpose than the cash flow C itself, so the manager's pay is based on this statistic.[10]

4 Optimal Contracts if the Manager can Trade

We now modify the previous analysis by assuming that the manager can trade in the stock market. More precisely, for the purpose of this subsection we assume that the manager can trade on the stock market index, for

[10] Note that the linearity of the information aggregation (the statistic $C - Ar_M$ is linear in C and r_M) would also obtain in a more general setting without imposing linearity of the contract; See Banker and Datar (1989).

example through trades in index futures. We will see later that we obtain different results if we relax this assumption and assume that the manager can actually trade in the individual components of the index. For the entire paper we assume that the manager can never trade in the stock of her own company, and that all trades take place in a perfect capital market with zero transaction costs and infinite liquidity. Now, in addition to receiving a salary from the firm, the manager also invests the amount α_M in the stock market index. The manager's end of period wealth is therefore given by:

$$\begin{aligned} W &= Y + \alpha_M r_M \\ &= \alpha_0 + \alpha_P C + (\alpha_M - \alpha_B)\, r_M \end{aligned} \tag{12}$$

The analysis of the previous section can now be modified accordingly. Equation (7) is modified only with respect to the variance expression:

$$Var\,(W) = \alpha_P^2 A^2 \sigma^2 + 2\alpha_P A\,(\alpha_M - \alpha_B)\,\sigma_M^2 + (\alpha_M - \alpha_B)^2\,\sigma_M^2 \tag{13}$$

Observe that the manager may adjust her portfolio subsequent to signing the compensation contract. Hence, before we can solve for the optimal contract we need to solve for the manager's optimal portfolio strategy:

$$\alpha_M = \alpha_B - \alpha_P A \tag{14}$$

Substituting the expression for α_M back into the variance-equation:

$$Var\,(W) = \alpha_P^2 A^2 \sigma_F^2 \tag{15}$$

Using the participation constraint ($CE\,(W) = 0$) and substituting for the equilibrium effort level ($e = \alpha_P/k$) we obtain shareholders' objective as before as:

$$V = E\,(P) = c_0 A + \frac{\alpha_P}{k} - \frac{\alpha_P^2}{2k} - \frac{\rho}{2}\alpha_P^2 A^2 \sigma_F^2 \tag{16}$$

This leads to exactly the same result as the case with relative performance evaluation before, and the optimal value of α_P is still given by (8).[11] However, there is a significant difference here, since we did not obtain an optimal solution for the relative performance parameter α_B. Effectively, since (16) is independent of α_B, *any* solution for α_B can be optimal. The reason for this indeterminacy is immediate from equation (14). The manager optimally adjusts her investment in the market α_M exactly so as to offset any change in α_B. We have therefore proven:

Proposition 2 ***(Trade):*** *Assume that the manager can trade in the stock market index but not in the stock of her own company subsequent to signing the contract with shareholders. (i) Then shareholders' wealth is independent of the relative performance parameter α_B, and it is weakly optimal not to use any relative performance evaluation relative to the market index ($\alpha_B = 0$). (ii) The resulting allocation for shareholder value, managerial effort and the slope parameter α_P of the optimal contract is the same as if the manager could not trade and the optimal relative performance contract where chosen.*

The important implication here is that the standard principal-agent contract without relative performance evaluation ($\alpha_B = 0$) is actually an optimal contract. If the manager can trade in an asset that is perfectly correlated with the benchmark of a relative performance contract, then she will follow a portfolio strategy that optimally hedges her against the risks incurred through her compensation contract. As a result, no relative performance contract can improve on the manager's optimal hedging strategy. This result contrasts Campbell and Kracaw (1987) who find for their model that the optimal insurance strategy of the manager is different from that

[11] Note that equation (35) in the appendix is valid without modification.

preferred by shareholders. They assume that the manager can trade in the stock of her own company, which leads her to insure excessively against the risk in her own company. Moreover, their assumption that insurable risks are uncorrelated removes any meaningful relative performance evaluation in the contract.

The important insight of Proposition 2 is that at the margin the manager will always adjust her exposure to the risk in the benchmark portfolio optimally, and any additional exposure to the market index in her contract will be reflected in a commensurate reduction in exposure in her portfolio, so that her *total* exposure to the risk reflected in the benchmark is independent of her contract. Note that this argument assumes that the manager may have to short-sell the stock market index, and that short-selling is costless. If short-selling the market index, or, equivalently, selling stock index futures, is costly, then a relative performance contract may be optimal again as a costless substitute for costly short-selling.

This argument reflects the same logical structure as the Modigliani-Miller-theorem on optimal capital structure. Since the manager can perfectly replicate the relative performance contract, any such contract simply replaces "manager-made insurance" with "firm-made insurance," without generating any additional benefits. The model generates also slightly different comparative static results than the standard principal-agent model. The standard principal agent model predicts that managers in industries with a higher total volatility of returns have less high-powered incentives (see (8)). The relative performance model and the trading model developed in this section predict that the managers in industries with higher firm-specific risk have less high-powered contracts.[12] A regression that controls for beta-risk should be able to distinguish between the standard model and the trading model, but it cannot distinguish between the relative performance model and the trading model because they are observationally equivalent in this respect.

One limitation of the analysis of this subsection is the fact that it suggests that managers need to short-sell the market portfolio from (14). This somewhat implausible - and most probably counterfactual - result depends critically on the simplifying assumption that the expected market return and therefore the risk premium is zero. Assume for the remaining part of this subsection that $r_M \sim N\left(\pi, \sigma_M^2\right)$, so that the expected return on the market is $\pi > 0$. Then the analysis above can be easily modified to yield:[13]

$$\alpha_M = \frac{\pi}{\rho\sigma_M^2} + \alpha_B - \alpha_P A \tag{17}$$

For the sake of an illustration assume that the market risk premium is 6% and consider the case of a CEO who manages a company with an asset size of $1 billion.[14] Assume the CEO has a risk exposure of about $5 per $1000 of shareholder value, which gives $\alpha_P = 0.005$.[15] Furthermore, assume

[12] Aggarwal and Samwick (1999) test this hypothesis and find strong evidence for the inverse relationship between α_P and σ^2. However, since firm-specific risk is a large component of σ^2, their test does not distinguish between the trading model, the relative performance model, and the standard model respectively.

[13] Clearly, if we assume a positive market risk premium $\pi > 0$ then some of the previous analysis needs to be modified accordingly. Following the approach of Garen (1994) it can be shown that proposition 1 above does not change materially, and proposition 2 does not change at all.

[14] These are US billions, so 1 bn=1,000,000,000. In mid 1999 the median S&P 500 company had a market capitalization of $8bn, with range of $400m to about $450bn.

[15] See Murphy (1999) for statistics on this. He shows that α_P depends on company's size and industry. His values range from $1.22/$1000 (utilities) to $28.23 (S&P Small cap

$\sigma_M = 0.2$, $\alpha_B = 0$ and $\rho = 0.2$.[16] Then we obtain that the CEO invests \$7.5 million in risky assets, the contract lets her invest \$5 million in the stock of her own company, and the remaining \$2.5 million she invests in the market portfolio.

Clearly, this demonstration falls short of a complete calibration analysis. A comparison with the calibration approach of Haubrich and Popova (1998) shows why such an analysis may be problematic. They analyze a cross-section of companies and find that very low values of ρ (like 0.025) explain their data best. Then our example would give an investment in risky assets of \$60bn, most of which is in the market index portfolio. Hence, for all except the most wealthy CEOs these numbers imply that their optimal investment in the stock market is large, and their portfolio is implausibly highly levered.[17] In this light the challenge to our model is not that it predicts a short position in the stock market. Rather, the challenge is to reconcile the low level of risk aversion that seems to be required to rationalize the cross-sectional evidence with the higher risk aversion implied by plausible portfolios of the same CEOs.

The main reason for this failure lies probably in the assumption of constant absolute risk aversion, implying that there are no wealth effects. Then the dollar value invested in risky assets is the same for the CEO of a small company and for the CEO of a very large company. The large investments

industrials).

[16] See Haubrich (1994) and Haubrich and Popova (1998) for a discussion of plausible values for ρ. The second article finds that their dataset is best explained by values of ρ as low as 0.025.

[17] Evidently, each of the numbers assumed here can be challenged. However, even a low market risk premium (5%) together with high market volatility (30%) leads to a total investment in risky assets of \$22bn, which is most likely far in excess of most CEOs wealth.

by the CEOs of larger companies can then only be reconciled with a model with constant absolute risk aversion if the parameter for risk aversion is set very low. Hence, cross-sectional data would probably be better explained by assuming that absolute risk-aversion can vary across CEOs. We therefore use higher coefficients of absolute risk aversion for the example above.

5 Accounting Benchmarks

The analysis in the previous subsection rests critically on the fact that the manager can trade on an asset that is perfectly correlated with the benchmark measure chosen in a relative performance contract. This is realistic if the manager is evaluated relative to the stock market index or an industry stock index that are typically traded in liquid markets. However, the benchmark could also be an accounting measure, that it is not traded directly in a public market. Accounting benchmarks include for example the average return on assets or the average return on equity of other companies in the same industry. These will be correlated with market movements, although imperfectly.

We now assume that the manager is evaluated relative to some benchmark measure B, which is correlated with the random component of the cash flow ε, but is only imperfectly correlated with the stock market index r_M. Define by σ_{FB} the covariance between the firm-specific component ε_F and B, and by σ_{MB} the covariance between the market return and B. The benchmark B is distributed normal with zero mean and variance σ_B^2. Then the manager's wealth is now:

$$W = \alpha_0 + \alpha_P C + \alpha_M r_M - \alpha_B B \tag{18}$$

The certainty equivalent is given by (7) with variance expression:

$$\begin{aligned} Var(W) &= (\alpha_P A + \alpha_M)^2 \sigma_M^2 + (\alpha_P A)^2 \sigma_F^2 + \alpha_B^2 \sigma_B^2 \\ &\quad -2(\alpha_P A + \alpha_M)\alpha_B \sigma_{BM} - 2\alpha_B \alpha_P A \sigma_{BF} \end{aligned} \tag{19}$$

Using calculations analogous to those in previous sections we find the manager's optimal portfolio strategy

$$\alpha_M = \alpha_B \frac{\sigma_{BM}}{\sigma_M^2} - \alpha_P A \tag{20}$$

Comparing this expression with (14) shows that the manager still adjusts her portfolio if she is benchmarked, but only to the extent that the benchmark is correlated with the market index. Denote by ρ_{BM} the coefficient of correlation between the market return r_M and the benchmark B. Then the optimal benchmark can easily be found as:

$$\alpha_B = \alpha_P A \frac{\sigma_{BF}}{\sigma_B^2 \left(1 - \rho_{BM}^2\right)} \tag{21}$$

This expression contains an important message, since it shows that α_B is zero whenever the covariance between the benchmark and *firm-specific* risk is zero. The extent to which the benchmark picks up movements in the stock market index r_M is irrelevant, since the trading strategy of the manager neutralizes the market-related component of the benchmark. Also, note that expression (21) reproduces the analysis of the previous subsection in a more general form: If the benchmark is perfectly correlated with the market index, then $\rho_{BM} = 1$ and the expression for α_B becomes infinite, so the relative performance component becomes indeterminate again.[18]

We can analyze expression (21) further by applying insights from standard regression analysis. Note that it is always possible to decompose the

[18] Then the optimal portfolio strategy of the manager (20) coincides with (14) again.

benchmark B into a component related to the index, and another component denoted by ε_{BO} that is orthogonal to any variation in the index:

$$B = \frac{\sigma_{BM}}{\sigma_M^2}\varepsilon_M + \varepsilon_{BO} \quad \varepsilon_{BO} \sim N\left(0, \sigma_B^2\left(1 - \rho_{BM}^2\right)\right) \tag{22}$$

where $Cov\left(\varepsilon_{BO}, \varepsilon_M\right) = 0$. Now denote by ρ_{OF} the coefficient of correlation between ε_F and ε_{BO}, the component of B that is orthogonal to the market. Then we can write the optimal contract as:

$$\alpha_P^A = \frac{1}{1 + \rho k A^2 \sigma_F^2\left(1 - \rho_{OF}^2\right)} \tag{23}$$

where the superscript A denotes the case where the relative performance contract includes an accounting benchmark. Note that $Cov\left(\varepsilon_O, \varepsilon_F\right) = \sigma_{BF}$ since ε_F is orthogonal to the market return by construction. Also, α_P^A exceeds α_P^R if and only if $\rho_{OF} > 0$, and therefore only if $\sigma_{BF} > 0$. Clearly, we obtain the same solution as we did previously (see (8)) if $\rho_{OF} = \sigma_{BF} = 0$. Hence, we have demonstrated the following result:

Proposition 3 ***(Accounting Benchmark):*** *Relative performance evaluation increases α_P and therefore improves the allocation if and only if the performance benchmark is correlated with firm-specific risk. If the accounting benchmark is correlated with market risk, but not with firm-specific risk, then $\alpha_B = 0$.*

The important part of this proposition is the insight that the conditions that must be satisfied so that an accounting benchmark becomes part of the optimal managerial contract are very restrictive. We can discuss the implication of Proposition 3 again by reference to another example. Suppose that our manager is the CEO of a construction company. Than we can clearly devise an optimal contract by benchmarking her against the average earnings (or some other accounting measure) of the construction industry in order to remove any influences from her compensation package that are not under her control as they affect the construction industry as a whole. However, construction is a cyclical industry, and the profits of other firms in the construction industry will be correlated - although imperfectly - with the market index. Then Proposition 3 says that benchmarking this manager against an index of earnings in the construction industry will not improve the contract to the extent that movements in the industry's earning index can be explained (in a regression sense) by movements in the stock market index. The contract is improved only if there is an industry-specific risk component that is common to all firms in the construction industry, but unrelated to macroeconomic risks.

Proposition 3 does help us to understand why accounting benchmarks are more widely used than portfolio benchmarks. Murphy (1998) summarizes the empirical evidence on relative performance evaluation (RPE) by writing: "I document the explicit use of RPE in accounting-based bonus plans, and discuss the virtual absence of RPE in stock option plans." (p. 5).

Note that the benchmark B could be a measure other than an accounting benchmark, for example it could be the stock market index of firms in the same industry, which would pick up market movements as well as some firm-specific components that are not related to the market return. Then the above result still obtains with the caveat that the manager is not permitted to trade in any asset other than the stock market index. The next section relaxes the restriction that the manager can only trade in the stock market index.

6 Unrestricted Portfolio Strategies

Assume the manager could also trade in the stocks of her competitors and firms in the same industry individually. Then we can demonstrate a slightly stronger result than Proposition 3 above. Now there are N assets the manager can trade in the market. We continue to assume that the manager cannot trade in the stock of her own firm. Denote by r_i the return on asset i, by α_i the manager's investment in asset i, by Σ the covariance matrix of all assets and by Σ_{BM}, Σ_{FM} and α the $N \times 1-$column-vectors of covariances with the benchmark, covariances with the firm-specific component and the portfolio weights α_i, respectively. Moreover, denote by ω_i the weight of asset i in the market portfolio, so that $\sum_{i=1}^{N} \omega_i = 1$, and by r and ω the $N \times 1-$column-vectors of returns and weights in the market portfolio. To preserve consistency with our previous analysis we obtain:[19]

$$\begin{aligned} \omega' r &= \varepsilon_M \\ \omega' \Sigma \omega &= \sigma_M^2 \\ \omega' \Sigma_{FM} &= 0 \end{aligned} \tag{24}$$

The analysis of this case involves some matrix manipulations that do not enhance the exposition and are therefore deferred to the appendix. We proceed by distinguishing two cases. The first case has again the market return as a benchmark (see section 4), whereas the second case uses an accounting benchmark (see section 5).

[19] Here an in the following primes of vectors denote a transpose. All vectors are defined as column-vectors.

6.1 Evaluation against a market benchmark

Assume for the purpose of this subsection that the manager is benchmarked against an index B that is composed entirely of the N assets in the market index, where β is the $N \times 1-$column-vector of weights in these assets, so we can write $B = \beta' r$ and $\sigma_B^2 = \beta' \Sigma \beta$. Then the optimal portfolio weights are given by:

$$\alpha = -\alpha_P A\omega + \alpha_B \beta - \alpha_P A \Sigma'_{FM} \Sigma^{-1} \tag{25}$$

Note that this expression parallels equation (20) above, with one important difference. We find again that the manager reduces her investment in the market index to the extent that she is already exposed to macroeconomic risk through her compensation. Also, she increases her investment in the assets of the market portfolio to the extent that they are correlated with the benchmark. However, there is no parallel in (20) for the third expression in (25). The reason is that while $\Sigma'_{FM}\omega = 0$, this does not imply that $\Sigma_{FM} = 0$ element by element. Reconsider the example of the manager of a construction company in the previous section: if there exists an industry-specific risk component (for example, related to the weather) that is not related to the macroeconomic shock, then the respective elements of Σ_{FM} are positive to reflect this industry risk component. Hence, the manager will reduce her investment in the assets of companies in her own industry. Clearly, even in a CAPM-world the manager will not hold the market portfolio. Since her compensation contract forces her to assume some firm-specific risk, she will bias her investments away from the market portfolio by investing less in those companies similar to her own, and investing more in companies that are correlated with her performance benchmark.

It follows from (25) that the composition of the benchmark index is unimportant, as long as the manager can trade on all components of the benchmark. If she can trade on all components of the index, then she neutralizes the benchmark by offsetting transactions in the market. Her wealth is still given by $W = Y + \alpha' r$, where Y is the income received from the company. Then:

$$W = \alpha_0 + \alpha_P \left(e + c_0 A\right) + \alpha_P A \left(\varepsilon_F - \Sigma'_{FM} \Sigma^{-1} r\right) \tag{26}$$

which is independent of the weight α_B the benchmark has in her compensation contract. Following the same procedures as in previous sections, we find the optimal contract as:

$$\alpha_P^{UM} = \frac{1}{1 + \rho k A^2 \sigma_F^2 \left(1 - R_{F,M}^2\right)} \tag{27}$$

where $R_{F,M}^2 = \Sigma'_{FM} \Sigma^{-1} \Sigma_{FM} / \sigma_F^2$ is the R-squared of the projection of the firm-specific component ε_F on the vector of market returns r.[20] We can therefore infer immediately:

Proposition 4 *If the manager can trade in all assets except the stock of her own company, then the value of the firm is (i) independent of the weight of the benchmark α_B in the contract, (ii) independent of the composition β of the benchmark, and (iii) strictly higher than the value of the company with an optimal relative performance contract where the manager is evaluated in comparison with the market index and the manager cannot trade if at least*

[20] To pursue the regression analogy further, note that the variance of the manager's wealth in equilibrium is $Var(W) = (\alpha_P A)^2 Var(u)$, where $u = \varepsilon_F - \Sigma'_{FM} \Sigma^{-1} r$ is the residual of the regression of the firm-specific component ε_F on the vector r of asset returns, and the vector $\Sigma^{-1}\ \Sigma_{FM}$ is the vector of regression coefficients in this regression. Clearly, in the special case where $\Sigma_{FM} = 0$, we obtain $u = \varepsilon_F$ and $R_{F,M}^2 = 0$, and we obtain again the solution of section 4.

one component of the return-vector r is correlated with firm-specific risk so that $\Sigma_{FM} \neq 0$*.*

The first two observations follow directly from the manager's wealth-equation (26). Part (iii) follows immediately from comparing (27) with (8). Hence, if trading by the manager is unrestricted then her equilibrium trading strategy will yield a *better* allocation than that achieved with a relative performance contract unless the relative performance contract employs the optimal benchmark. This benchmark is derived below.

We can now turn the argument presented so far around and ask what the optimal composition of the benchmark is in an environment where the manager cannot trade. Then it is easy to show:

Proposition 5 ***(Optimal Benchmark)*** *Assume the manager is not allowed to trade and does not hold any risky assets in her portfolio. Then it is optimal to benchmark the manager against a portfolio* β *so that:*

$$\alpha_B \beta = \alpha_P A \left(\omega + \Sigma^{-1} \Sigma_{FM}\right) \tag{28}$$

Then the optimal contract parameter α_P *is given by (27) and the value of the firm is exactly equal to the value obtained if there is no performance benchmark and the manager is unrestricted in her trading strategies.*

This result is a direct implication of the informativeness principle, and shows clearly the defect of benchmarking the manager against the market portfolio ω. Benchmarking against the market portfolio is optimal only if $\Sigma_{FM} = 0$, so that the firm-specific risk is uncorrelated with all other tradeable assets. (see the condition in Proposition 4, part (iii)).However, if there exists an industry-specific risk-component that is shared by a number of firms, but not by the economy as a whole then these correlations need to be incorporated in the optimal benchmark. Similarly, if the firm's cash flows depend on prices that are linked to derivatives traded on public exchanges (e. g., commodities or currencies), then again these assets should be included in the optimal benchmark. However, this argument holds only if the manager cannot trade in these assets, and shows that the optimal contract should not impose any restrictions on the manager's portfolio strategy. The manager's trading strategy does not only neutralize any benchmark composed of tradeable assets, it is also a perfect substitute for finding the *optimal* benchmark.

6.2 Evaluation against an accounting benchmark

For the purpose of this subsection we change only the assumption that the benchmark B can be written as some combination of traded assets. Note that the benchmark may still be a vector of several accounting and other measures, and the optimal composition of this benchmark can be found by following steps similar to those in the previous subsection. There is no need to repeat this analysis here, and we can therefore treat B as a scalar.[21] Denote by $Var\left(B\left|r\right.\right)$ the conditional variance of the benchmark B, conditional on security returns r, and, similarly, by $Cov\left(B, \varepsilon_F\left|r\right.\right)$ the conditional covariance between B and firm-specific risk ε_F. Then the optimal

[21] Assume there is an $M \times 1-$vector of candidate benchmark components m. Then the optimal linear contract can be written as an $M \times 1-$vector of weights γ. Then the above analysis effectively uses $\alpha_B = \beta' 1$ and $B = \gamma' m / \gamma' 1$, where 1 denotes the $M \times 1-$vector of ones.

weight of the benchmark is:

$$\alpha_B^{UA} = \frac{Cov\left(B, \varepsilon_F \left| r\right.\right)}{Var\left(B \left| r\right.\right)} \alpha_P A \tag{29}$$

where the superscript 'U' denotes the case where the manager's trading opportunities are not restricted. This result parallels (21): the optimal contract has a role for a benchmark only to the extent that the benchmark is correlated with the firm-specific component, conditional on the returns of all securities the manager can trade. Finally, the optimal contract can now be expressed as:

$$\alpha_P^U = \frac{1}{1 + \rho k A^2 \sigma_F^2 \left(1 - R_{F,BM}^2\right)} \tag{30}$$

where $R_{F,BM}^2$ denotes the R-squared of the regression of ε_F on the benchmark B and the vector of security returns r. Note that the last expression exactly parallels equation (23).

In order to interpret this result we follow the same steps as in the previous section and decompose B and ε_F as follows:

$$B = \Sigma_{BM}' \Sigma^{-1} r + \varepsilon_{BO} \qquad \varepsilon_{BO} \sim N\left(0, \sigma_B^2 - \Sigma_{BM}' \Sigma^{-1} \Sigma_{BM}\right) \tag{31}$$

$$\varepsilon_F = \Sigma_{FM}' \Sigma^{-1} r + \frac{Cov\left(B, \varepsilon_F \left| r\right.\right)}{Var\left(B \left| r\right.\right)} \varepsilon_{BO} + \eta \qquad \eta \sim N\left(0, \sigma_\eta^2\right) \tag{32}$$

Hence, we extend our regression analogy from the previous section by projecting the risk of the cash flow on the vector of security returns and on the benchmark. In order to make this analogy more transparent, we use the equivalent procedure of projecting ε_F on r and the component of B that is orthogonal to r. Then direct calculation shows that:

$$\sigma_\eta^2 = Var\left(W\right) = \sigma_F^2 \left(1 - R_{F,BM}^2\right) \tag{33}$$

We can therefore immediately observe the following result:

Proposition 6 ***(Unrestricted Trading):*** *Assume the manager can trade on all securities in the market portfolio. Then the optimal contract will make use of relative performance evaluation only if the conditional covariance between the firm-specific component in the firm's cash flow and the benchmark (where conditioning is with respect to all tradeable security returns) is different from zero.*

This result extends Holmström's informativeness principle in the following way. Holmström shows that the optimal principal-agent contract will use an additional variable in the benchmark if and only if this variable contributes additional information with respect to all other variables already included in the contract. Proposition 6 shows that the additional variable must have incremental information also with respect to all returns of securities the manager can trade on *in addition to* the variables included in the contract. Hence, even if the benchmark contributes new information to the other variables in the contract (the unconditional covariance is zero: $Cov\,(B, \varepsilon_F) \neq 0$), this information may already be contained in the returns of assets the manager can trade on ($Cov\,(B, \varepsilon_F\,|r\,) = 0$).

In order to illustrate Proposition 6, consider a firm in the gold-mining industry. Clearly, we can evaluate the performance of the manager of this firm by using some profit measure, and then using the average profits of other companies in the gold-mining industry as a benchmark. However, a major influence on the profits of all firms in the industry clearly comes from the gold price itself, and gold futures are traded on several derivatives exchanges. Assume the manager can trade on two assets: the stock market index and the gold price. Then Proposition 6 states that this benchmark is part of an optimal contract only if the following three conditions are met:

1. The firm-specific risk of the firm in question is correlated with the firm-specific risk of other firms in the gold-mining industry, so that returns of firms in the gold-mining firms have a common, industry-specific component.

2. This industry-wide component must be different from general macroeconomic risk.

3. This industry-wide component is not correlated with the gold-price itself.

If there is an industry-specific risk-component which is orthogonal to the market index, and orthogonal to the gold price, then benchmarking the manager against average industry profits will remove this component from her compensation contract. Conditions 1. and 2. are already part of the analysis of previous sections, whereas the third condition is the additional insight behind Proposition 6.

The preceding argument borrows from standard multiple regression analysis. This allows us to rephrase the insights generated in all the subsections of this discussion so far. We can state the following corollary of Proposition 6:

Corollary 7 ***(Regression)*** *The benchmark B is part of an optimal compensation contract if and only if B adds explanatory power to a regression where the random component of the manager's performance measure is regressed on all the securities the manager is allowed to trade on.*

In the previous discussion, the condition in corollary 7 follows directly from (29) and (30) above: if $R^2_{F,BM}$ (the R-squared from regressing the cash flows of the firm on the security returns r and the benchmark B) exceeds R^2_{FM} (the R-squared from regressing the cash flows of the firm on the security returns r alone), then the benchmark has incremental "explanatory power." Clearly, this is equivalent to the requirement that $Cov\left(\varepsilon_{FO}, \varepsilon_{BO}\right) \neq 0$, but much stronger than the requirement that the benchmark is correlated with the risk inherent in the manager's performance measure.

Finally, we can use this framework also in order to assess the usefulness of trading restrictions on the manager. If $R^2_{F,M} > 0$, then $\sigma^2_F \left(1 - R^2_{F,M}\right) < \sigma^2_F$ and shareholder value increases. Hence, to the extent that firm-specific risk is correlated with some other tradeable assets, the total risk borne by the manager in equilibrium is reduced through her trading:

Corollary 8 ***(Welfare of trading)*** *If the manager is allowed to trade on all assets except her own stock, then the resulting allocation is improved (that is, shareholder value is higher) relative to a situation where the manager can only trade on the stock market index, or where she cannot trade at all and is benchmarked against the stock market index.*

Hence, to the extent that the trading policies of the manager can also reduce the firm-specific risk - as in the case of the manager of the gold-mine hedging the risk in the gold price - benchmarking the manager against the index is inferior to allowing the manager to trade. Preventing the manager from trading strictly reduces overall welfare.

7 Conclusion

This paper analyzes a model where a manager is offered a compensation contract that may include some relative performance element. The main conclusion is that relative performance evaluation is of limited - if any - use in an environment where the manager can trade on all assets except the stock of her own company. The manager's trading strategy exactly replicates the optimal relative performance contract that would otherwise be proposed by the principal. If the manager is allowed to trade in all components of the index, then her trading strategy also replicates the composition of the optimal benchmark the principal would chose. The manager simply adjusts her portfolio strategy to offset any increased exposure she incurs through her compensation contract. The result is an argument that parallels that of Modigliani and Miller (1958): The manager provides "home-made" relative performance elements if they are not part of the contract, and reduces her exposure to macroeconomic risks by reducing her holdings in the market portfolio to the extent that her contract exposes her to market risk. Any relative performance element in the contract will therefore be offset by substituting "firm-made" benchmarking for the manager's "home-made" benchmark. The argument does not fully extend to accounting benchmarks if these have components that are not related to traded assets. Benchmarking the manager against non-traded assets is useful if it reduces the manager's exposure to risks that cannot be neutralized through trading.

The results presented here help to explain why there is little relative performance evaluation in most compensation contracts, and why most relative performance elements observed in these contracts are related to accounting benchmarks. The argument presented here has also some political implications for the discussion on executive compensation, since it suggests that rising managerial salaries may be a normal implication of a rising stock market, even though it may appear to contradict standard economic theory to reward even mediocre performance. Clearly, relative performance contracts for

CEOs of mediocre or underperforming companies would lead to lower payouts from their variable pay-component. However, if the market for CEOs is competitive, then these contracts would have a comparatively higher fixed components, which these same CEOs would invest in the stock market as a whole to end up with the same wealth (and the same compensation costs to the company) as they do without a relative performance contract. Hence, in the context of this model, relative performance evaluation cannot help to mitigate the rise of executive compensation.

8 Appendix

Derivation of (8), (9):

Observe that the participation constraint for the manager is binding ($CE(W,e) = 0$), so we obtain the fixed component of the compensation package as:

$$\alpha_0 = -\alpha_P c_0 A - \frac{\alpha_P^2}{2k} + \frac{\rho}{2} Var(W) \tag{34}$$

Then the payoff V to shareholders can be computed as:

$$V = E(P) = c_0 A + \frac{\alpha_P}{k} - \frac{\alpha_P^2}{2k} - \frac{\rho}{2}\left[\alpha_P^2 A^2 \sigma^2 - 2\alpha_P A \alpha_B \sigma_M^2 + \alpha_B^2 \sigma_M^2\right] \tag{35}$$

This gives the solution for the optimal weight of the benchmark as

$$\alpha_B = \alpha_P A \tag{36}$$

Substituting the solution for α_B back gives:

$$V = c_0 A + \frac{\alpha_P}{k} - \frac{\alpha_P^2}{2k} - \frac{\rho}{2}\alpha_P^2 \sigma_F^2 \tag{37}$$

Now we can solve for α_P to obtain (8).

Derivation of (25):

Use the same approach as before and observe that the variance of the manager's wealth can now be written as:

$$Var(W) = \begin{pmatrix} \alpha_P A\omega + \alpha - \alpha_B \beta \\ \alpha_P A \end{pmatrix}' \begin{pmatrix} \Sigma & \Sigma_{FM} \\ \Sigma'_{FM} & \sigma_F^2 \end{pmatrix} \begin{pmatrix} \alpha_P A\omega + \alpha - \alpha_B \beta \\ \alpha_P A \end{pmatrix} \tag{38}$$

Minimizing this expression with respect to α_M gives (25). Substituting back into (38) gives:

$$\begin{aligned} Var(W) &= (\alpha_P A)^2 \left(\sigma_F^2 - \Sigma'_{FM}\Sigma^{-1}\Sigma_{FM}\right) && (39) \\ &= (\alpha_P A)^2 \sigma_F^2 \left(1 - R_{F,M}^2\right) && (40) \end{aligned}$$

which then leads to (27) using the same steps as before.

Proof of Proposition 5:

Use expression (38) above with $\alpha = 0$ and then minimize with respect to $\alpha_B \beta$ to obtain the result.

Derivation of the optimal contract (29), (30) with unrestricted trading:

These expressions simply restate more formally the definition of ω as the market portfolio. Then we can rewrite the variance expression as:

$$Var\left(W\right) = \begin{pmatrix} \alpha_P A\omega + \alpha \\ \alpha_P A \\ -\alpha_B \end{pmatrix}' \begin{pmatrix} \Sigma & \Sigma_{FM} & \Sigma_{BM} \\ \Sigma'_{FM} & \sigma_F^2 & \sigma_{FB} \\ \Sigma'_{BM} & \sigma_{FB} & \sigma_B^2 \end{pmatrix} \begin{pmatrix} \alpha_P A\omega + \alpha \\ \alpha_P A \\ -\alpha_B \end{pmatrix} \tag{41}$$

The manager chooses portfolio weights:

$$\alpha = -\alpha_P A\omega + \Sigma^{-1}\left(\Sigma_{BM}\alpha_B - \Sigma_{FM}\alpha_P A\right) \tag{42}$$

Substituting α back into the variance expression gives then:

$$\begin{aligned} & Var\left(W\right) + \alpha \\ = & \begin{pmatrix} \Sigma^{-1}\left(\Sigma_{BM}\alpha_B - \Sigma_{FM}\alpha_P A\right) \\ \alpha_P A \\ -\alpha_B \end{pmatrix}' \begin{pmatrix} \Sigma & \Sigma_{FM} & \Sigma_{BM} \\ \Sigma'_{FM} & \sigma_F^2 & \sigma_{FB} \\ \Sigma'_{BM} & \sigma_{FB} & \sigma_B^2 \end{pmatrix} \\ & \times \begin{pmatrix} \Sigma^{-1}\left(\Sigma_{BM}\alpha_B - \Sigma_{FM}\alpha_P A\right) \\ \alpha_P A \\ -\alpha_B \end{pmatrix} \end{aligned} \tag{43}$$

$$= \begin{pmatrix} \Sigma_{BM}\alpha_B - \Sigma_{FM}\alpha_P A \\ \alpha_P A \\ -\alpha_B \end{pmatrix}' \begin{pmatrix} \Sigma^{-1} & \Sigma^{-1}\Sigma_{FM} & \Sigma^{-1}\Sigma_{BM} \\ \Sigma^{-1}\Sigma'_{FM} & \sigma_F^2 & \sigma_{FB} \\ \Sigma^{-1}\Sigma'_{BM} & \sigma_{FB} & \sigma_B^2 \end{pmatrix}$$

$$\times \begin{pmatrix} \Sigma_{BM}\alpha_B - \Sigma_{FM}\alpha_P A \\ \alpha_P A \\ -\alpha_B \end{pmatrix} \tag{44}$$

$$= -\left(\Sigma_{BM}\alpha_B - \Sigma_{FM}\alpha_P A\right)' \Sigma^{-1} \left(\Sigma_{BM}\alpha_B - \Sigma_{FM}\alpha_P A\right)$$
$$+\alpha_P^2 A^2 \sigma_F^2 - 2\alpha_P A \alpha_B + \alpha_B^2 \sigma_B^2 \tag{45}$$

This gives the optimal benchmark as:

$$\alpha_B = \frac{\sigma_{FB} - \Sigma'_{FM}\Sigma^{-1}\Sigma_{BM}}{\sigma_B^2 - \Sigma'_{BM}\Sigma^{-1}\Sigma_{BM}} \alpha_P A = \frac{\sigma_{FB} - \Sigma'_{FM}\Sigma^{-1}\Sigma_{BM}}{\sigma_B^2 \left(1 - R_{BM}^2\right)} \alpha_P A \tag{46}$$

which leads immediately to (29). Here $R_{BM}^2 = \Sigma'_{BM}\Sigma^{-1}\Sigma_{BM}/\sigma_B^2$ is the R-squared of the projection of the benchmark B on the vector of security returns r. Substituting the result back into the variance expression:

$$Var\left(W\right)$$
$$= -\left(\alpha_P A\right)^2 \left\{ \left(\Sigma_{BM} \frac{\sigma_{FB} - \Sigma'_{FM}\Sigma^{-1}\Sigma_{BM}}{\sigma_B^2 \left(1 - R_{BM}^2\right)} - \Sigma_{FM} \right)' \right.$$
$$\times \Sigma^{-1} \left(\Sigma_{BM} \frac{\sigma_{FB} - \Sigma'_{FM}\Sigma^{-1}\Sigma_{BM}}{\sigma_B^2 \left(1 - R_{BM}^2\right)} - \Sigma_{FM} \right) \tag{47}$$
$$\left. +\sigma_F^2 - 2\frac{\sigma_{FB} - \Sigma'_{FM}\Sigma^{-1}\Sigma_{BM}}{\sigma_B^2 \left(1 - R_{BM}^2\right)} + \left(\frac{\sigma_{FB} - \Sigma'_{FM}\Sigma^{-1}\Sigma_{BM}}{\sigma_B^2 \left(1 - R_{BM}^2\right)} \right)^2 \sigma_B^2 \right\}$$

$$= \frac{\alpha_P^2 A^2}{\sigma_B^2 \left(1 - R_{BM}^2\right)} \left\{ \sigma_F^2 \sigma_B^2 - \sigma_{FB}^2 - \sigma_B^2 \Sigma'_{FM}\Sigma^{-1}\Sigma_{FM} \right.$$
$$\left. -\sigma_F^2 \Sigma'_{BM}\Sigma^{-1}\Sigma_{BM} + 2\sigma_{FB}\Sigma'_{FM}\Sigma^{-1}\Sigma_{BM} \right\} \tag{48}$$

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